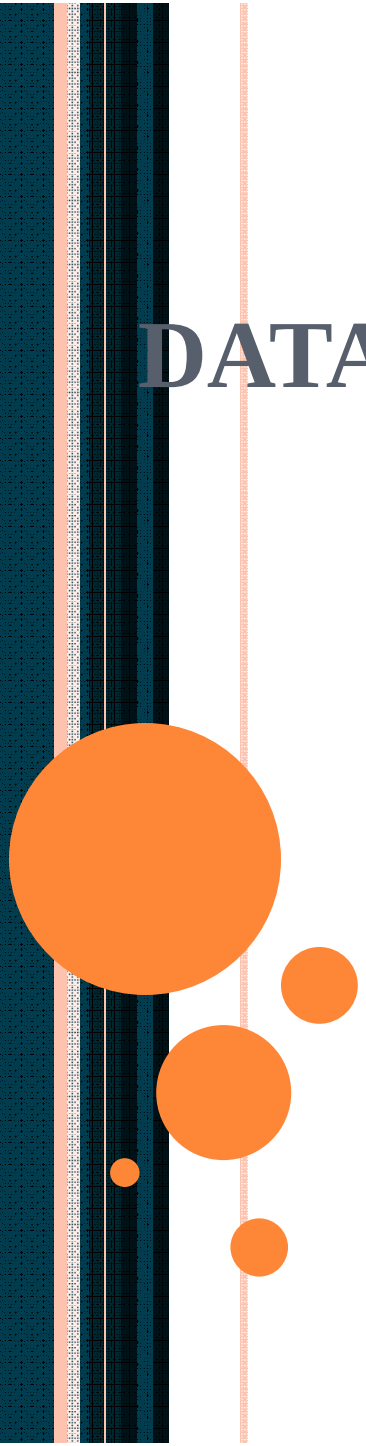
DATA STRUCTURES USING ‘C’



Lecture-12

Data Structures

Different types of Sorting Techniques used in Data Structures

Notes on Quicksort

- Quicksort is more widely used than any other sort.
- Quicksort is well-studied, not difficult to implement, works well on a variety of data, and consumes fewer resources that other sorts in nearly all situations.
- Quicksort is O(n*log n) time, and O(log n) additional space due to recursion.

Quicksort Algorithm

- Quicksort is a divide-and-conquer method for sorting. It works by partitioning an array into parts, then sorting each part independently.
- The crux of the problem is how to partition the array such that the following conditions are true:
  - `There is some element, a[i], where a[i] is in its final position.`
  - `For all l < i, a[l] < a[i].`
  - `For all i < r, a[i] < a[r].`

Quicksort Algorithm (cont)

- As is typical with a recursive program, once you figure out how to divide your problem into smaller subproblems, the implementation is amazingly simple.

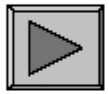
```
int partition(Item a[], int l, int r);
void quicksort(Item a[], int l, int r)
  { int i;
    if (r <= l) return;
    i = partition(a, l, r);
    quicksort(a, l, i-1);
    quicksort(a, i+1, r);
  }
```

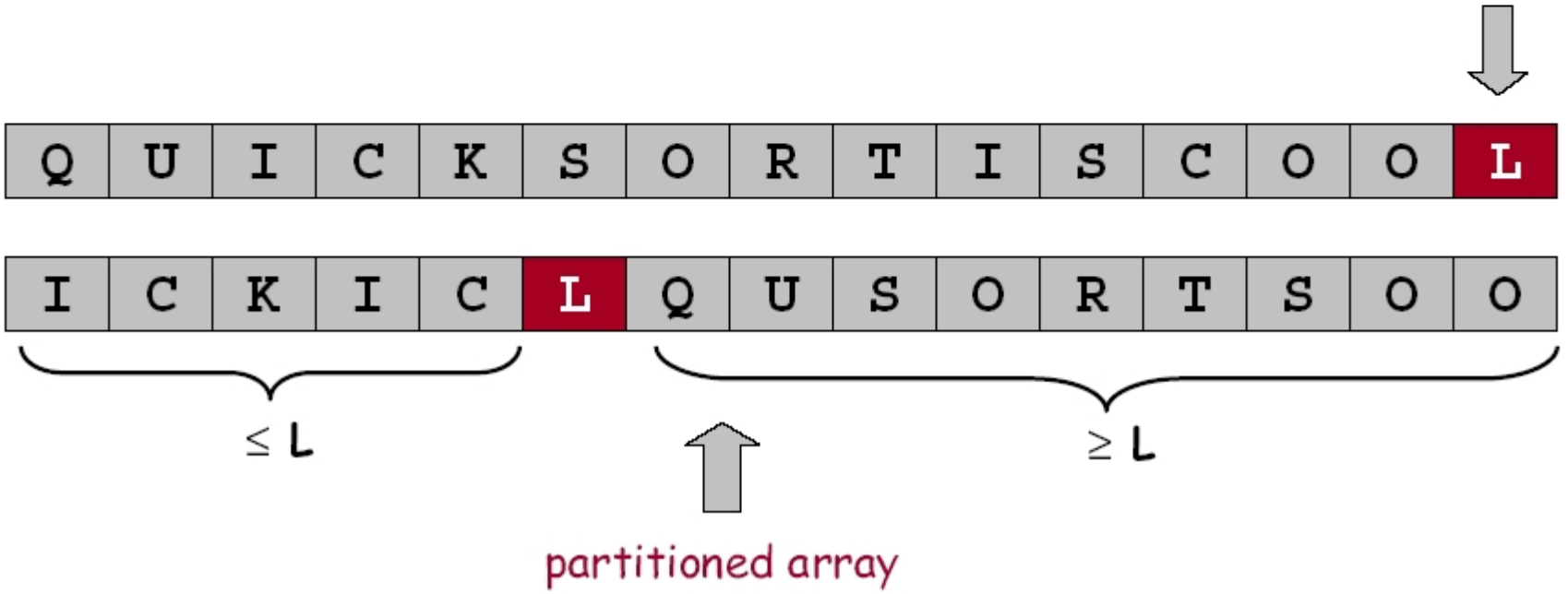
Quicksort

C. A. R. Hoare

Quicksort.

- Partition array so that:
  - some partitioning element `a[m]` is in its final position
  - no larger element to the left of `m`
  - no smaller element to the right of `m`

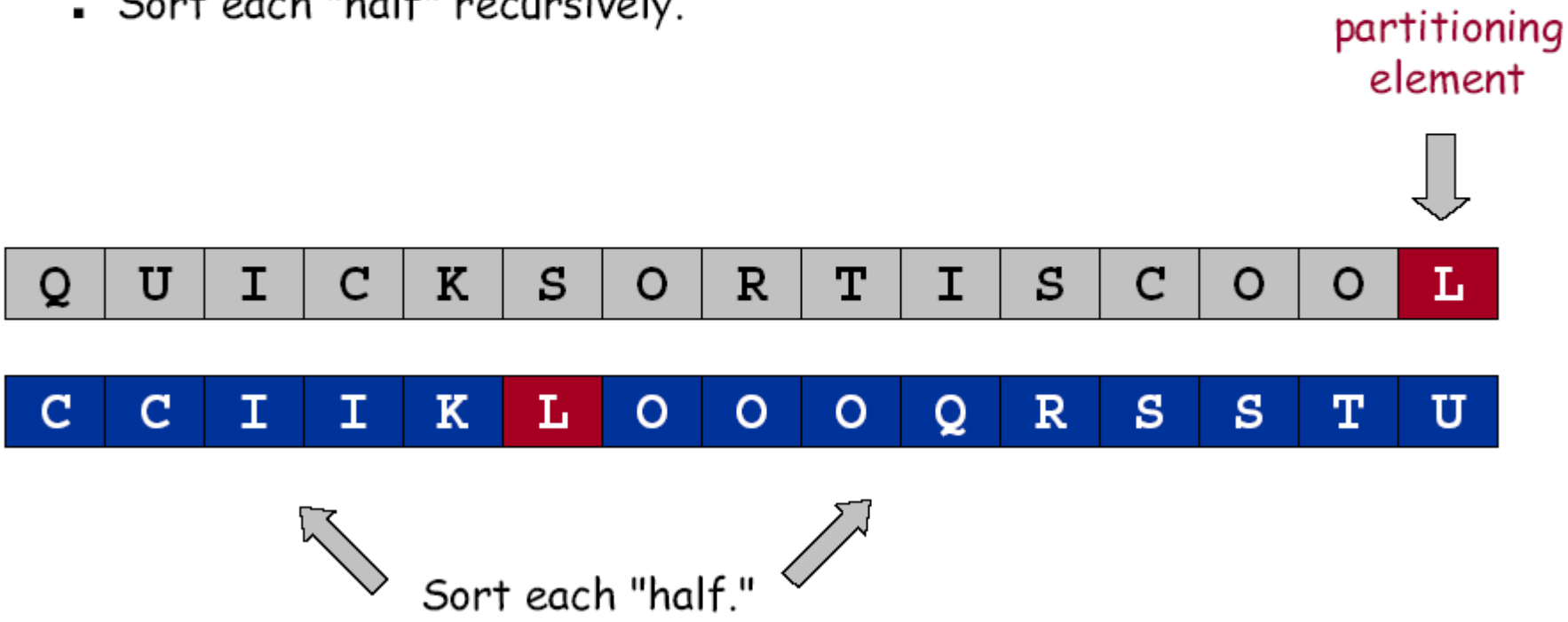




Quicksort

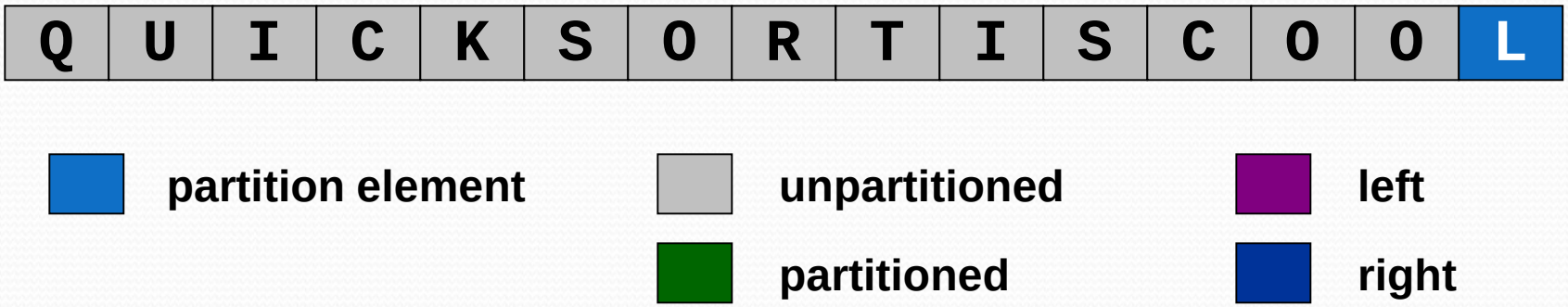
Quicksort.

- Partition array so that:
  - some partitioning element `a[m]` is in its final position
  - no larger element to the left of `m`
  - no smaller element to the right of `m`
- Sort each "half" recursively.



Partitioning in Quicksort

- How do we partition the array efficiently?
  - choose partition element to be rightmost element
  - scan from left for larger element
  - scan from right for smaller element
  - exchange
  - repeat until pointers cross



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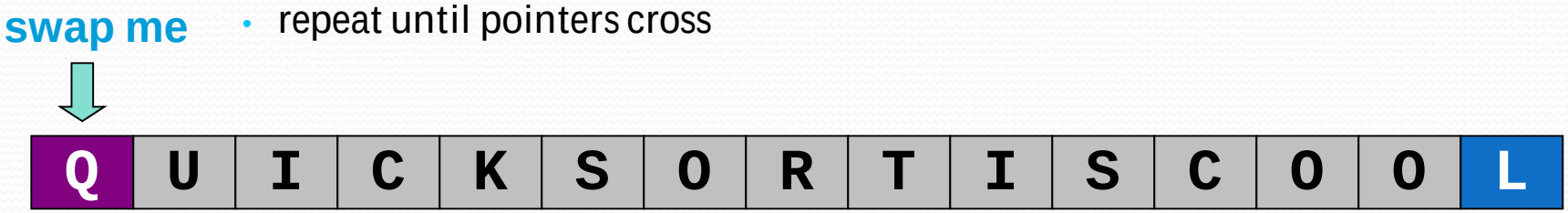

partition element

unpartitioned

left

partitioned

right

Partitioning in Quicksort

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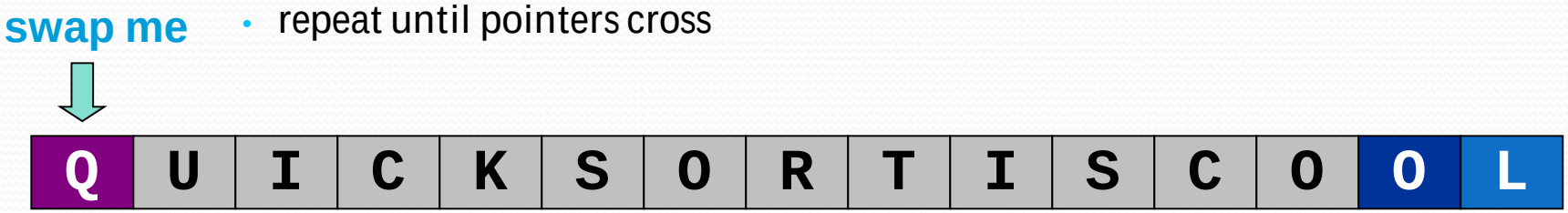

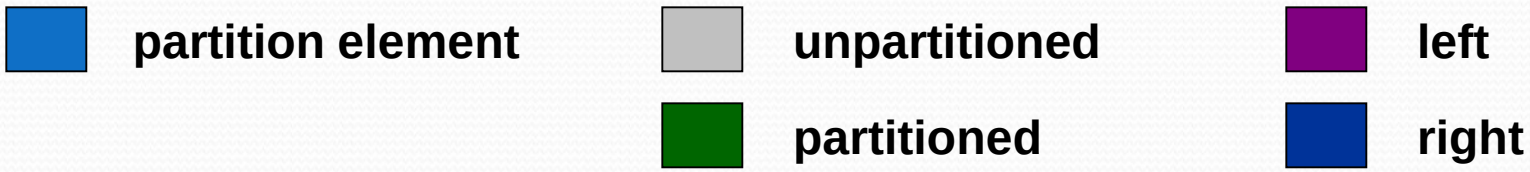

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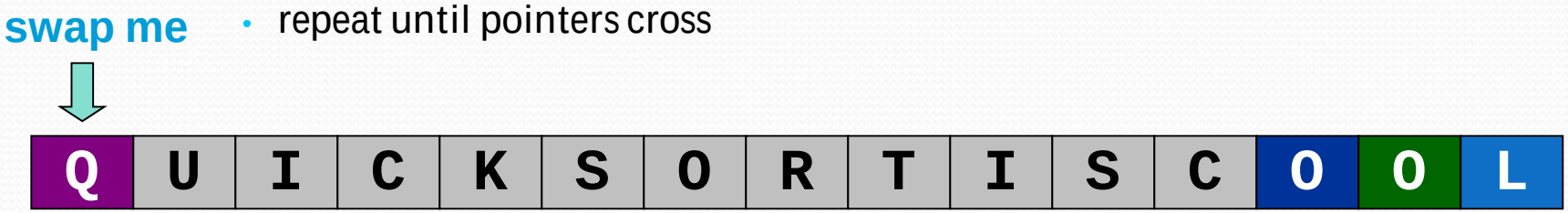

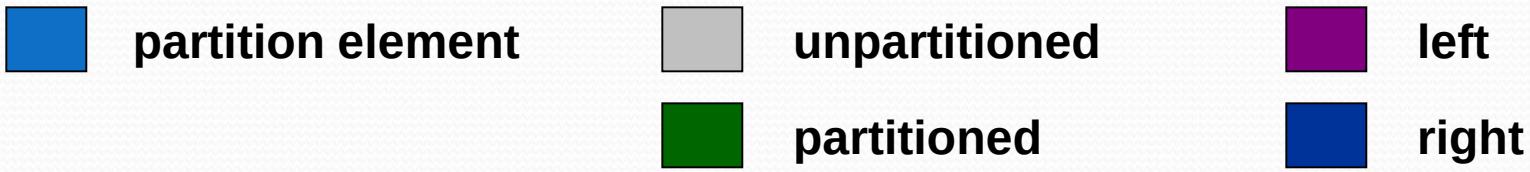

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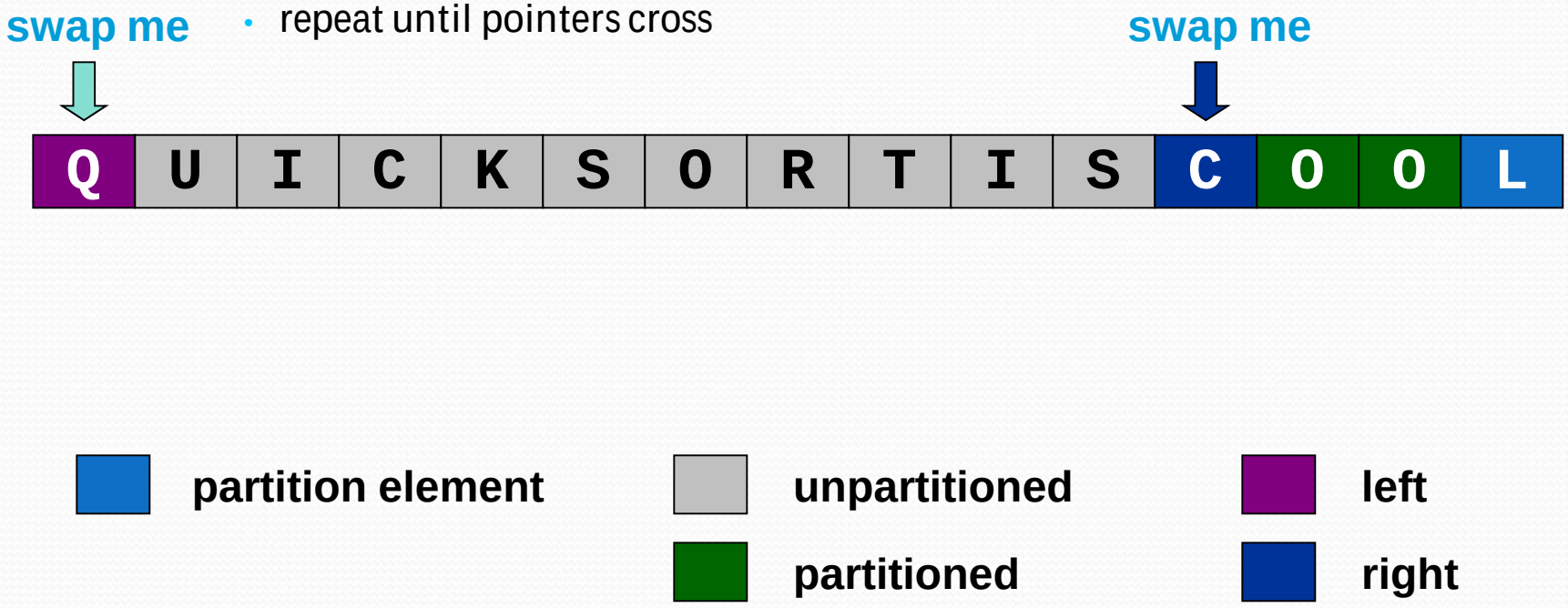

Partitioning in Quicksort

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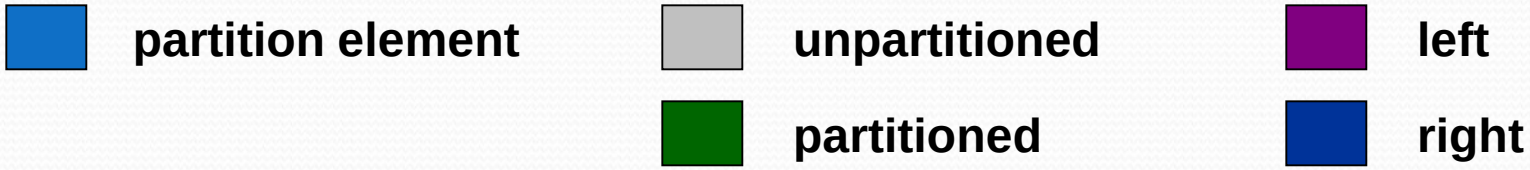

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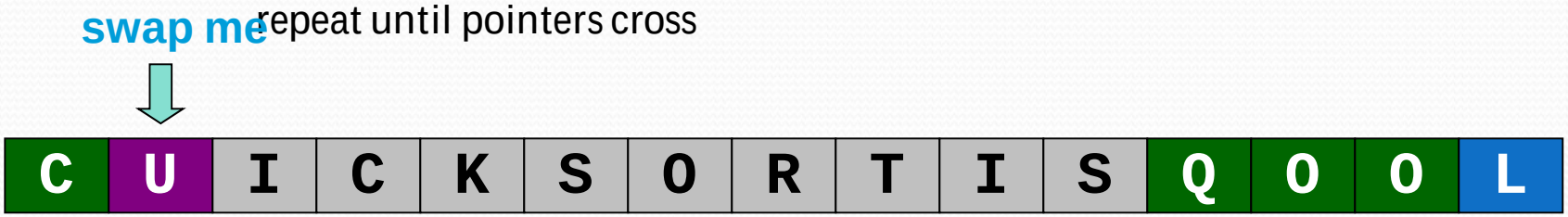

partition element

unpartitioned

left

partitioned

right

Partitioning in Quicksort

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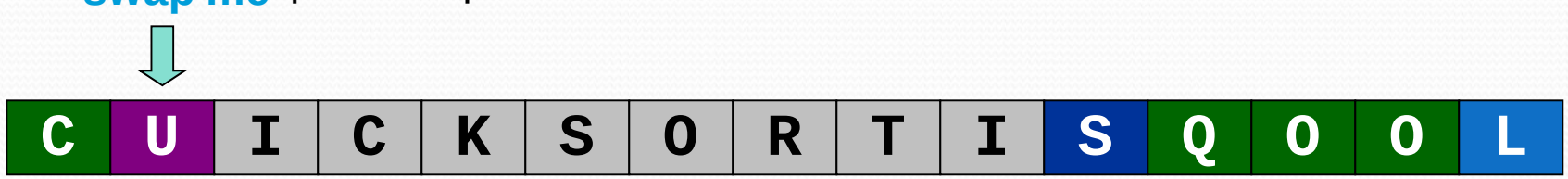

partition element

unpartitioned

left

partitioned

right

Partitioning in Quicksort

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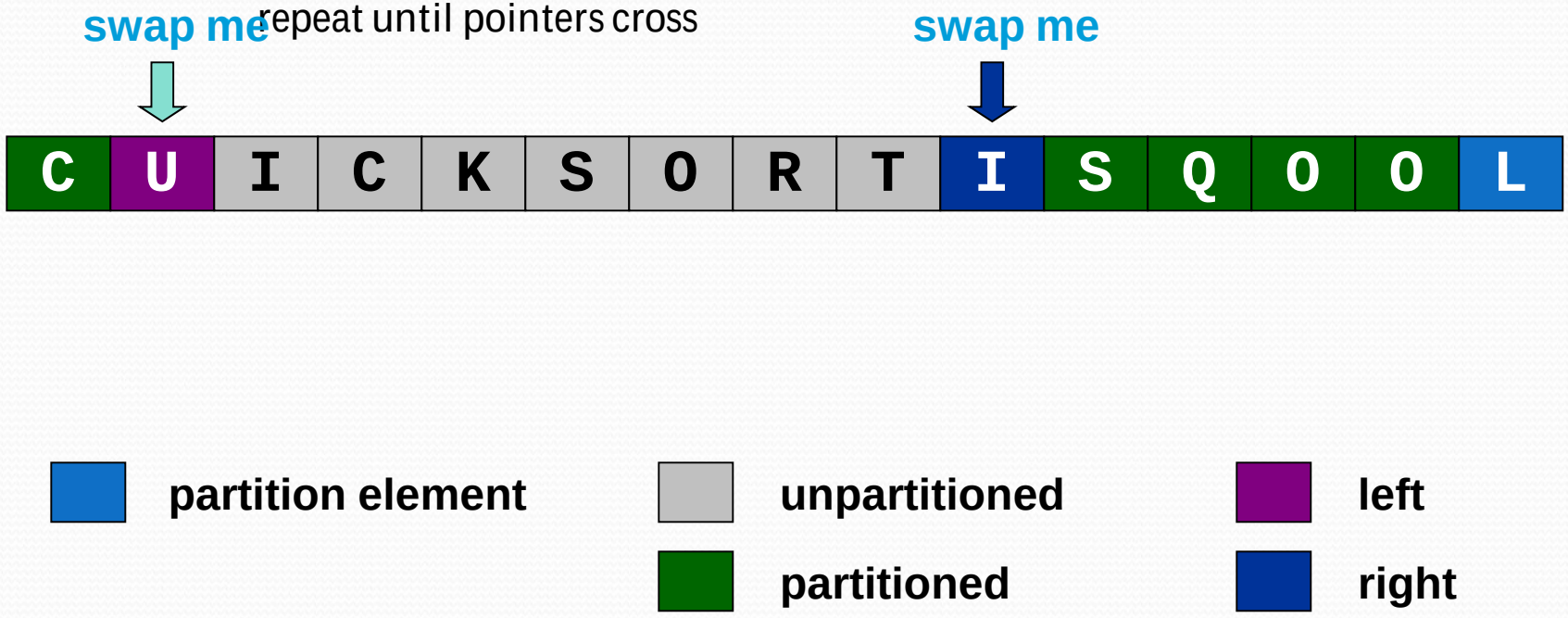

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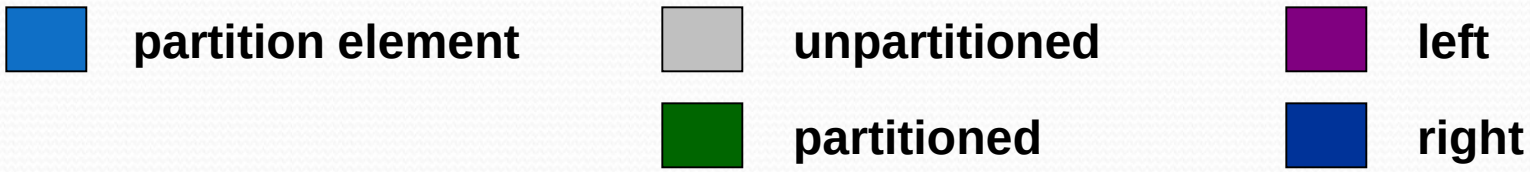

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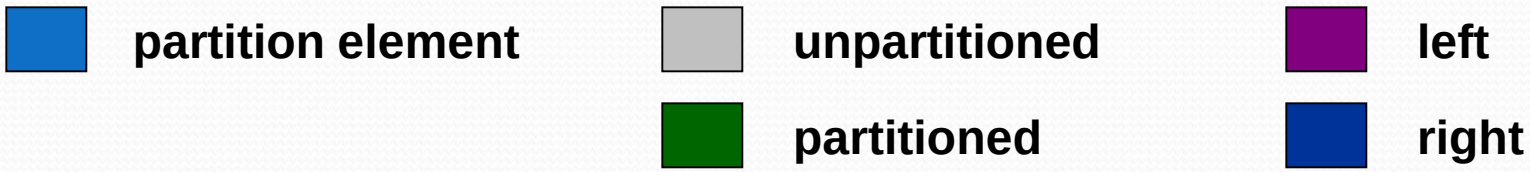

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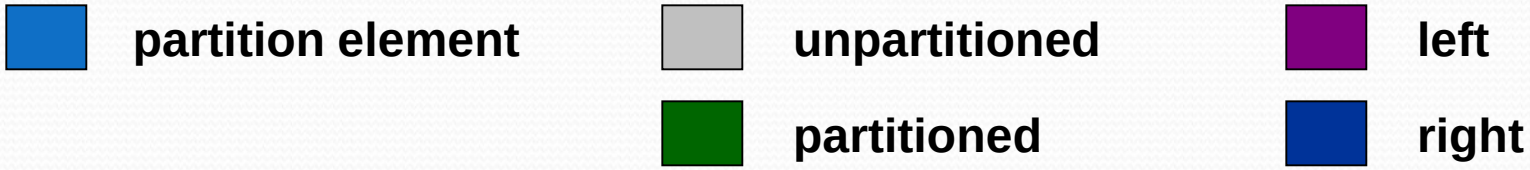

Partitioning in Quicksort

- How do we partition the array efficiently?
  - choose partition element to be rightmost element
  - scan from left for larger element
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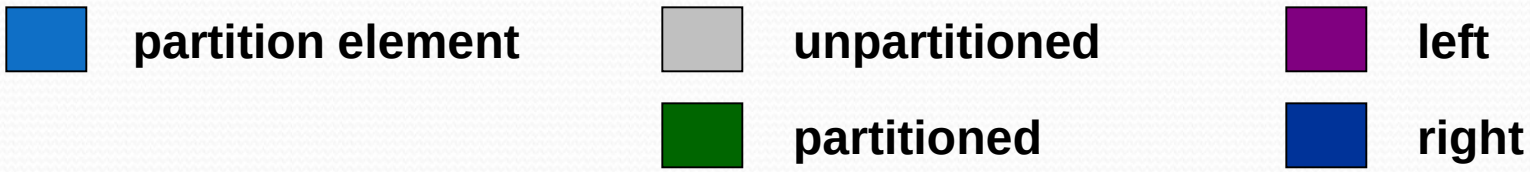
  - Exchange and repeat until pointers cross



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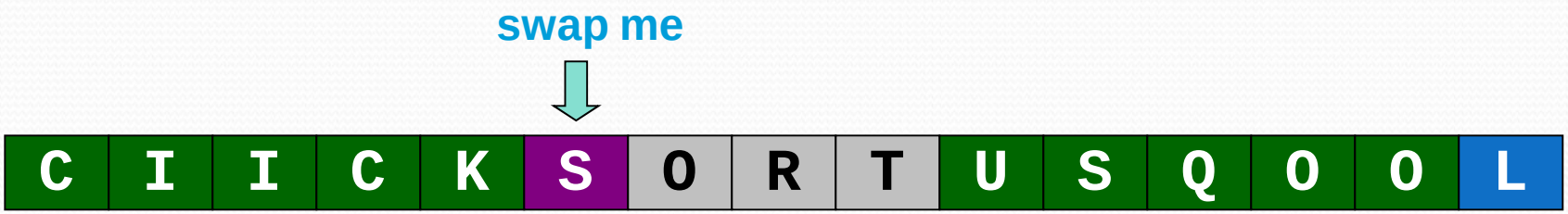

partition element

unpartitioned

left

partitioned

right

Partitioning in Quicksort

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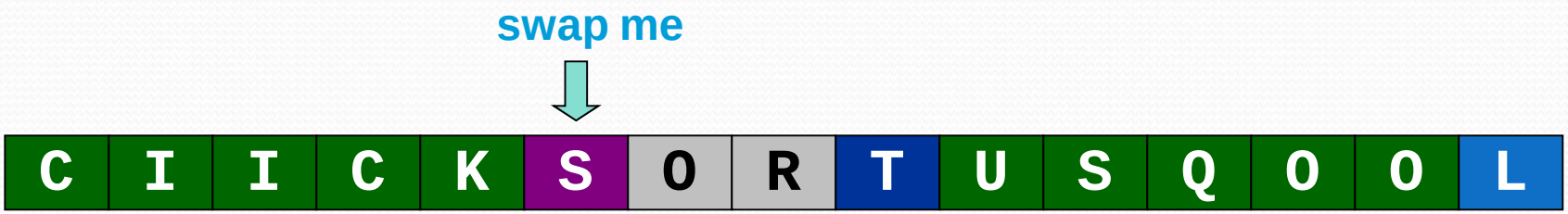

partition element

unpartitioned

left

partitioned

right

Partitioning in Quicksort

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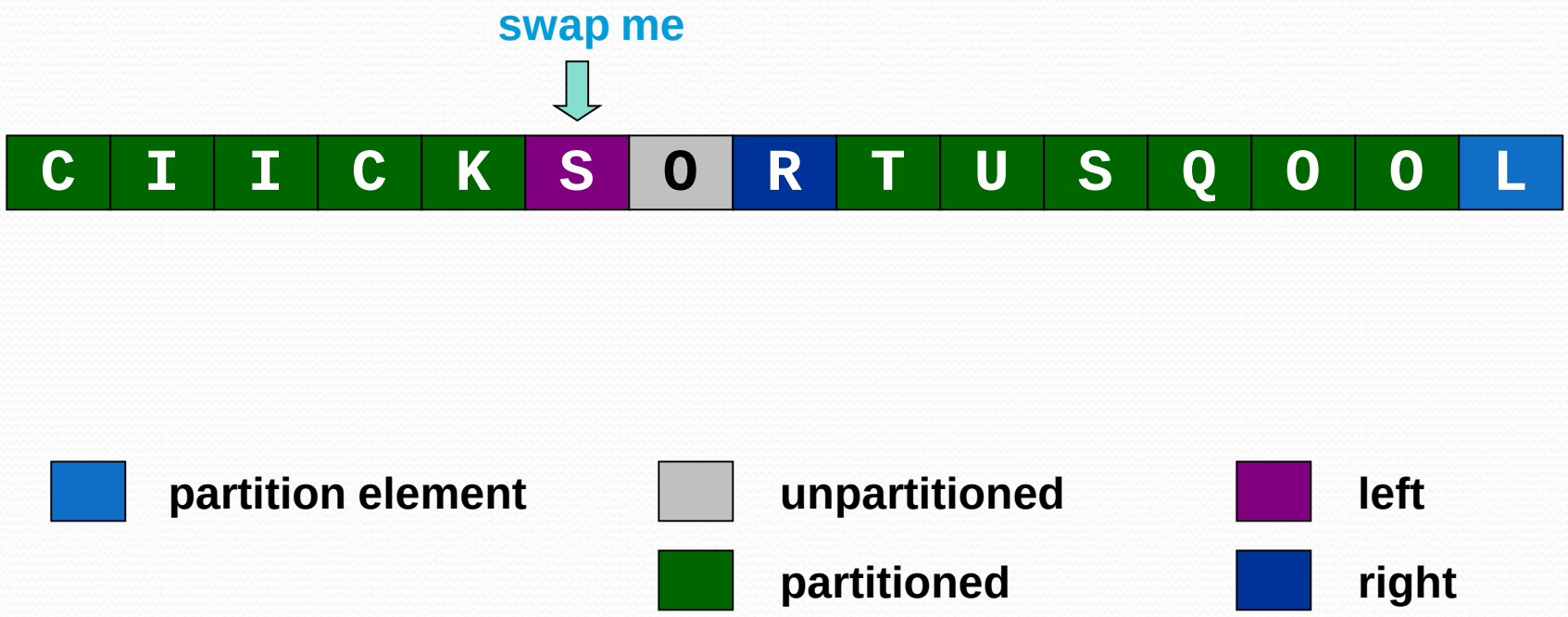

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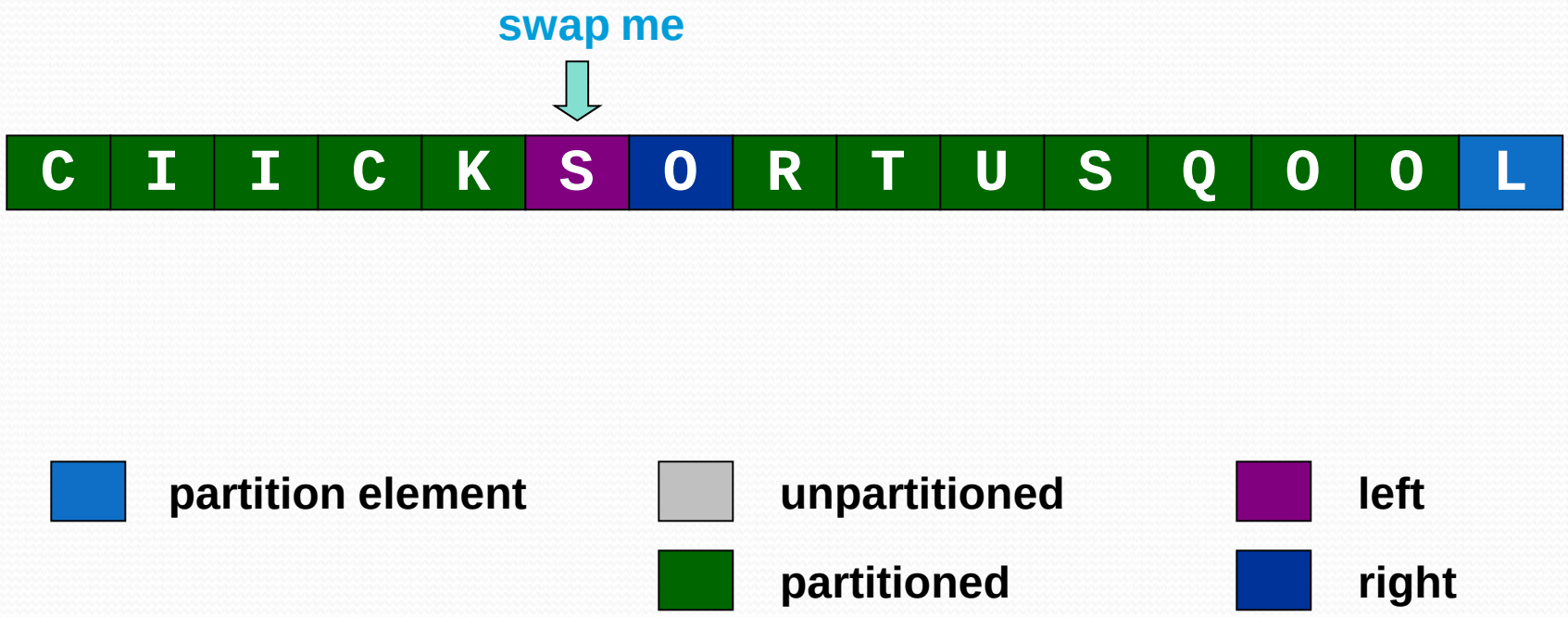


Partitioning in Quicksort

- How do we partition the array efficiently?
  - choose partition element to be rightmost element
  - scan from left for larger element
  - scan from right for smaller element
  - Exchange and repeat until pointers cross

pointers cross

swap with partitioning element

partition element

unpartitioned

left

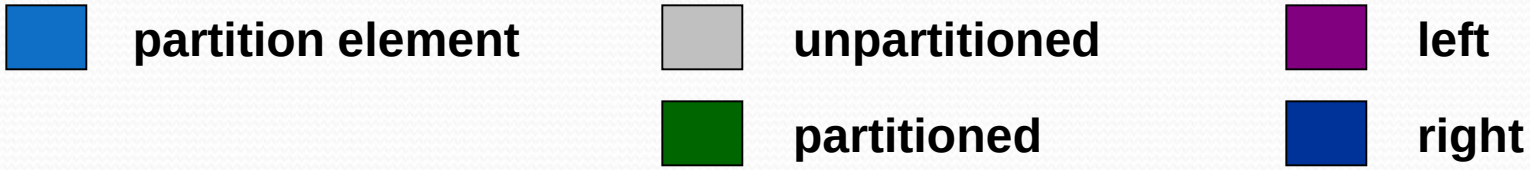
partitioned

right

Partitioning in Quicksort

- How do we partition the array efficiently?
  - choose partition element to be rightmost element
  - scan from left for larger element
  - scan from right for smaller element
  - Exchange and repeat until pointers cross

partition is complete



Quicksort Demo

- Quicksort illustrates the operation of the basic algorithm. When the array is partitioned, one element is in place on the diagonal, the left subarray has its upper corner at that element, and the right subarray has its lower corner at that element. The original file is divided into two smaller parts that are sorted independently. The left subarray is always sorted first, so the sorted result emerges as a line of black dots moving right and up the diagonal.

Why study Heapsort?

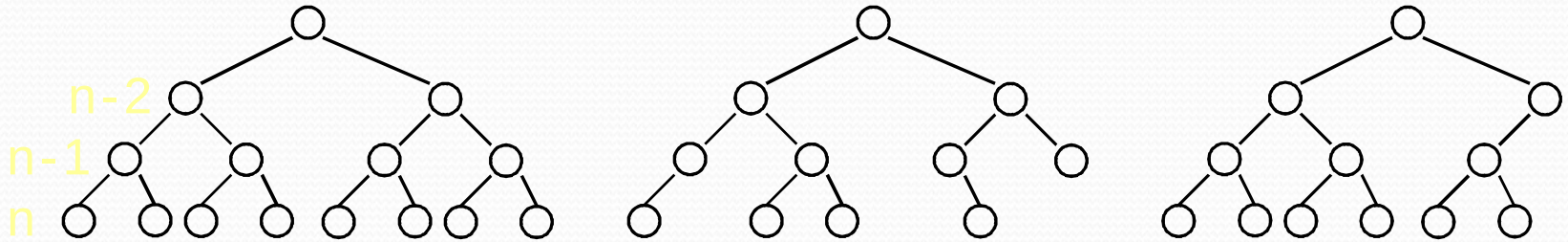
- It is a well-known, traditional sorting algorithm you will be expected to know
- Heapsort is *always* O(n log n)
  - Quicksort is usually O(n log n) but in the worst case slows to $O(n^2)$
  - Quicksort is generally faster, but Heapsort is better in time-critical applications

What is a “heap”?

- Definitions of heap:
  1. A large area of memory from which the programmer can allocate blocks as needed, and deallocate them (or allow them to be garbage collected) when no longer needed
  2. A balanced, left-justified binary tree in which no node has a value greater than the value in its parent
- Heapsort uses the second definition

Balanced binary trees

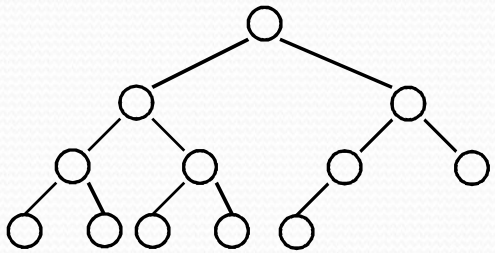
- Recall:
  - The depth of a node is its distance from the root
  - The depth of a tree is the depth of the deepest node
- A binary tree of depth n is balanced if all the nodes at depths 0 through n-2 have two children



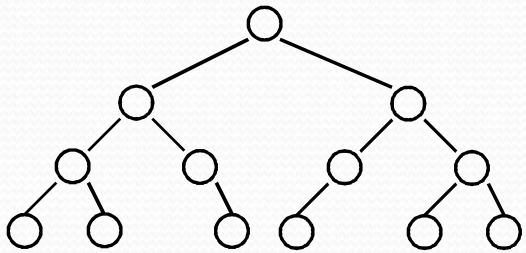
Balanced Balanced Not balanced

Left-justified binary trees

- A balanced binary tree is left-justified if:
  - all the leaves are at the same depth, or
  - all the leaves at depth $n+1$ are to the left of all the nodes at depth n



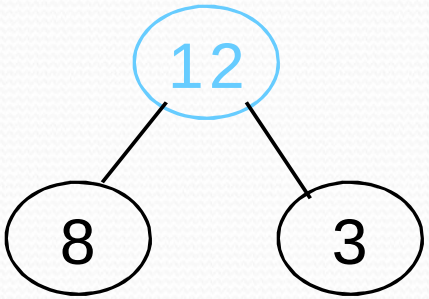
Left-justified



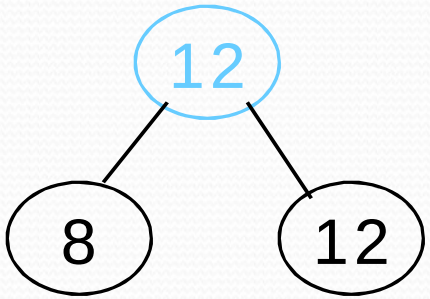
Not left-justified

The heap property

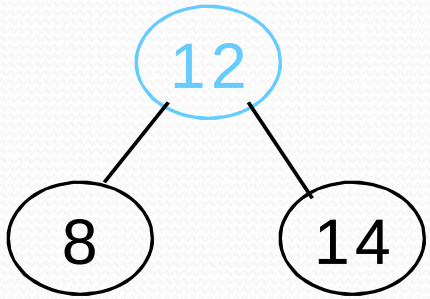
- A node has the heap property if the value in the node is as large as or larger than the values in its children



Blue node has heap property



Blue node has heap property

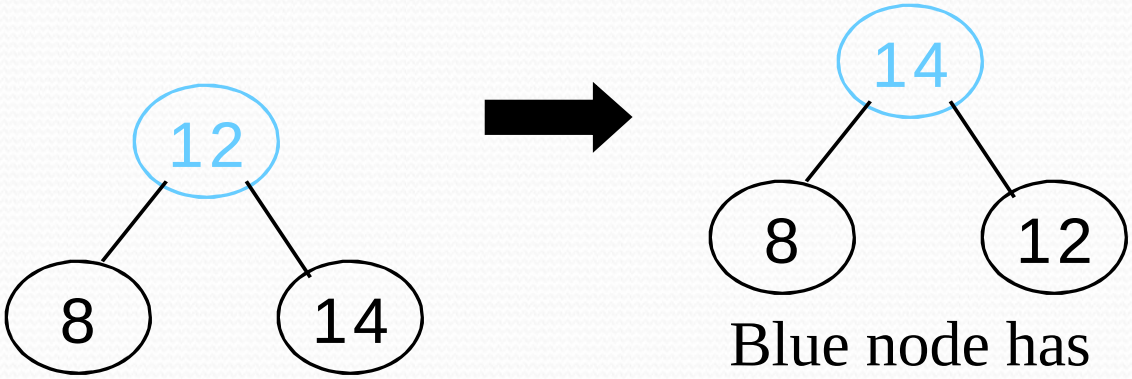


Blue node does not have heap property

- All leaf nodes automatically have the heap property
- A binary tree is a heap if *all* nodes in it have the heap property

siftUp

- Given a node that does not have the heap property, you can give it the heap property by exchanging its value with the value of the larger child



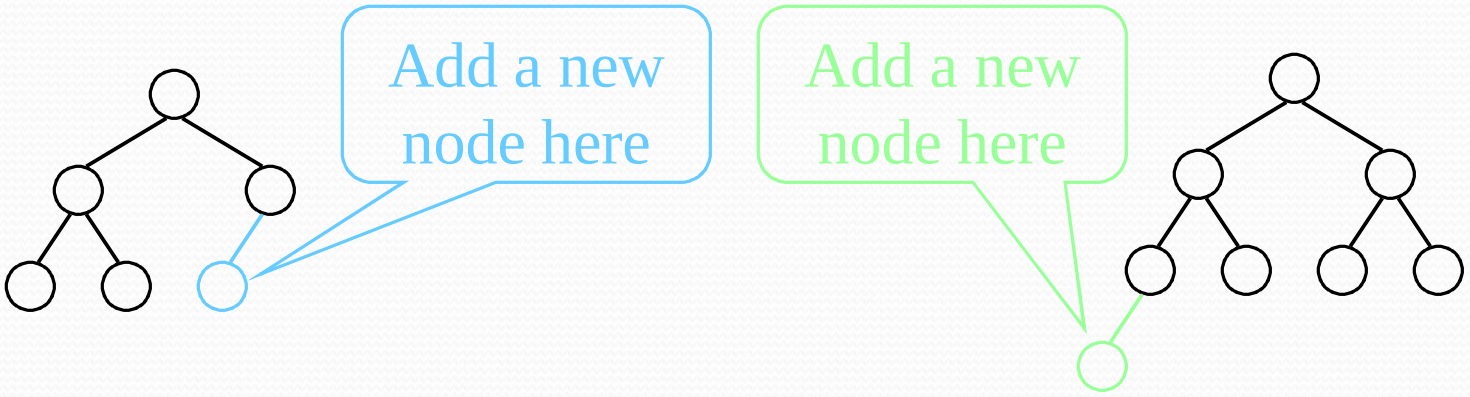
Blue node does not have heap property

Blue node has heap property

- This is sometimes called sifting up
- Notice that the child may have *lost* the heap property

Constructing a heap I

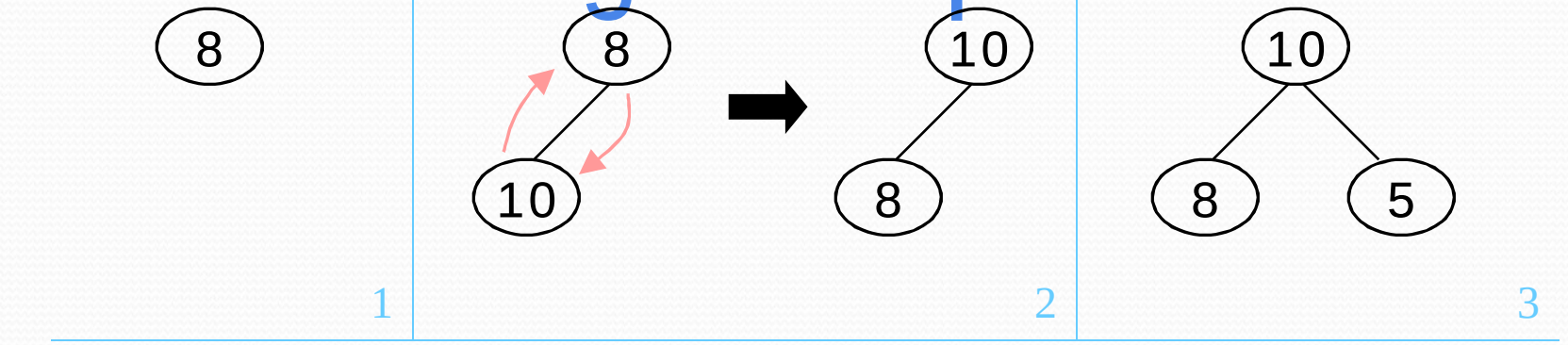
- A tree consisting of a single node is automatically a heap
- We construct a heap by adding nodes one at a time:
  - Add the node just to the right of the rightmost node in the deepest level
  - If the deepest level is full, start a new level
- Examples:

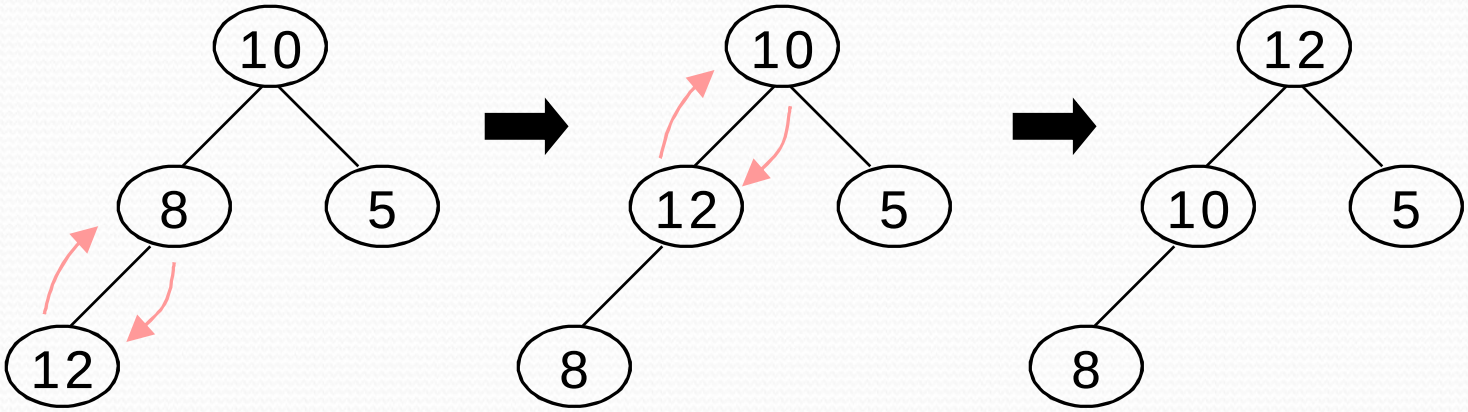


Constructing a heap II

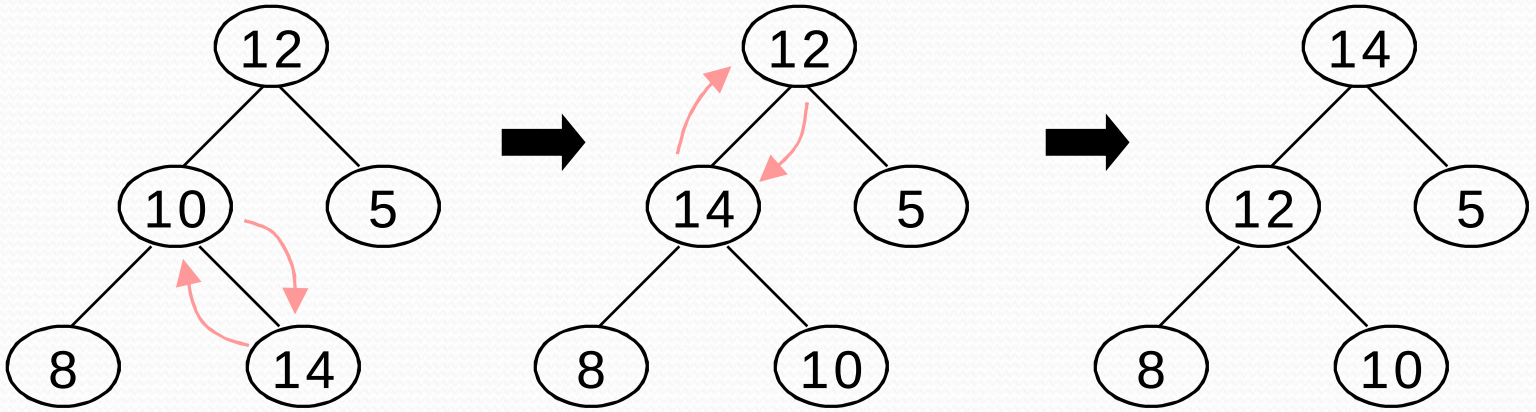
- Each time we add a node, we may destroy the heap property of its parent node
- To fix this, we sift up
- But each time we sift up, the value of the topmost node in the sift may increase, and this may destroy the heap property of *its* parent node
- We repeat the sifting up process, moving up in the tree, until either
  - We reach nodes whose values don't need to be swapped (because the parent is *still* larger than both children), or
  - We reach the root

Constructing a heap III





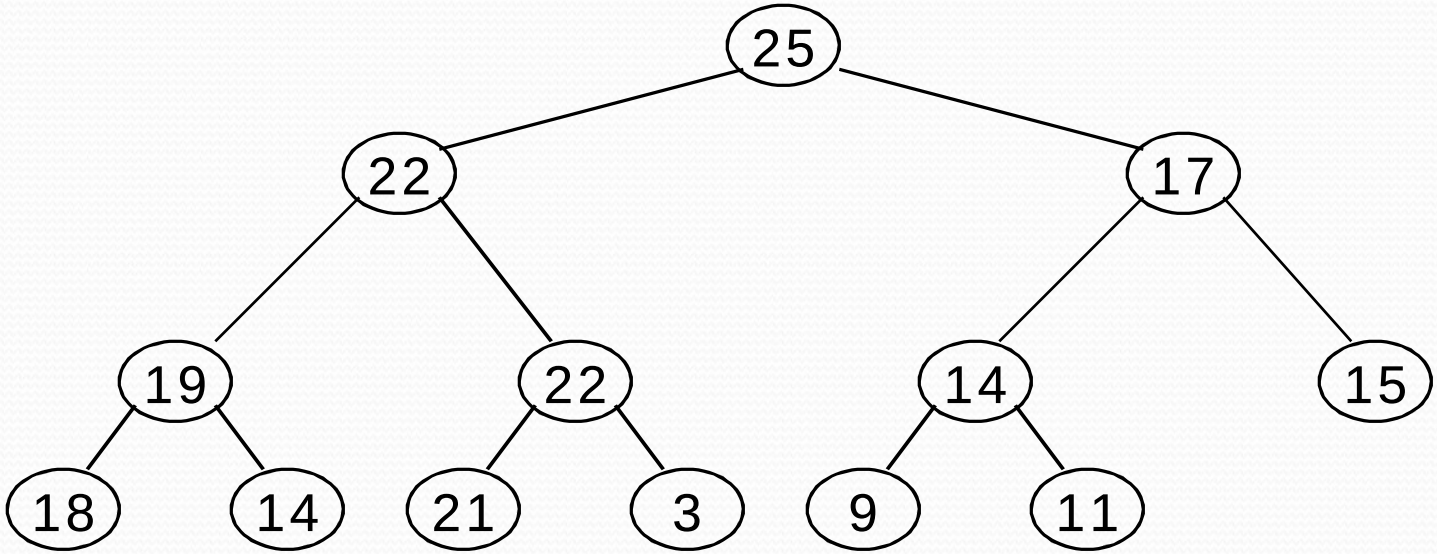
Other children are not affected



- The node containing 8 is not affected because its parent gets larger, not smaller
- The node containing 5 is not affected because its parent gets larger, not smaller
- The node containing 8 is still not affected because, although its parent got smaller, its parent is still greater than it was originally

A sample heap

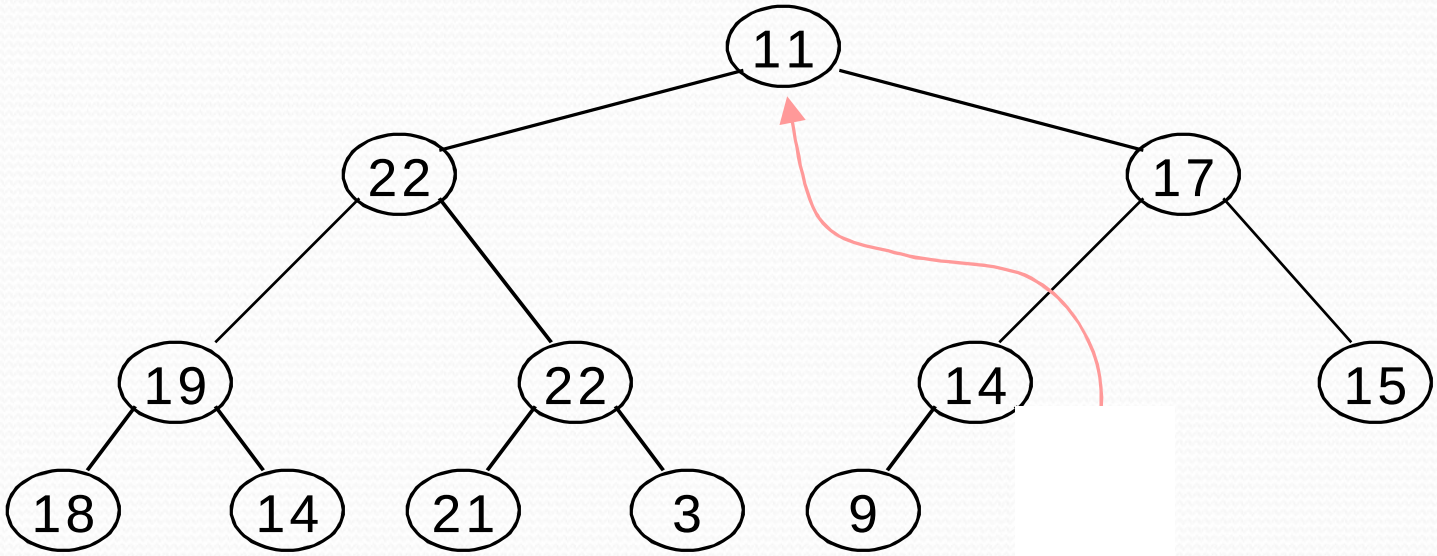
- Here's a sample binary tree after it has been heapified



- Notice that heapified does *not* mean sorted
- Heapifying does *not* change the shape of the binary tree; this binary tree is balanced and left-justified because it started out that way

Removing the root

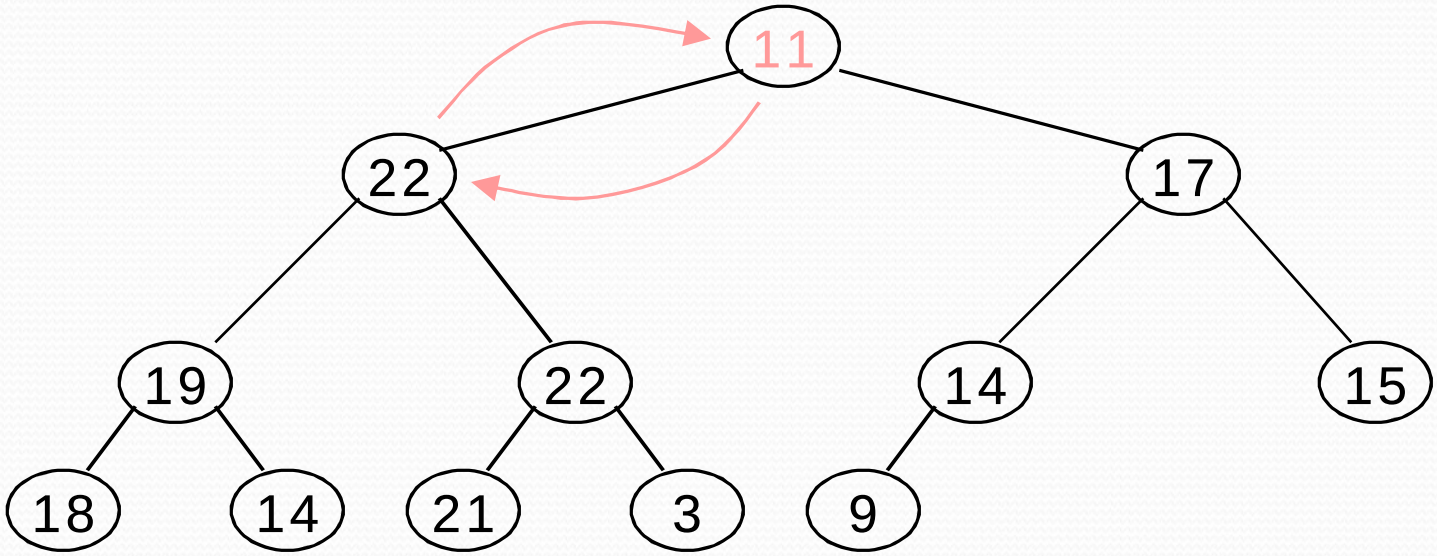
- Notice that the largest number is now in the root
- Suppose we *discard* the root:



- How can we fix the binary tree so it is once again *balanced and left-justified?*
- Solution: remove the rightmost leaf at the deepest level and use it for the new root

The reHeap method I

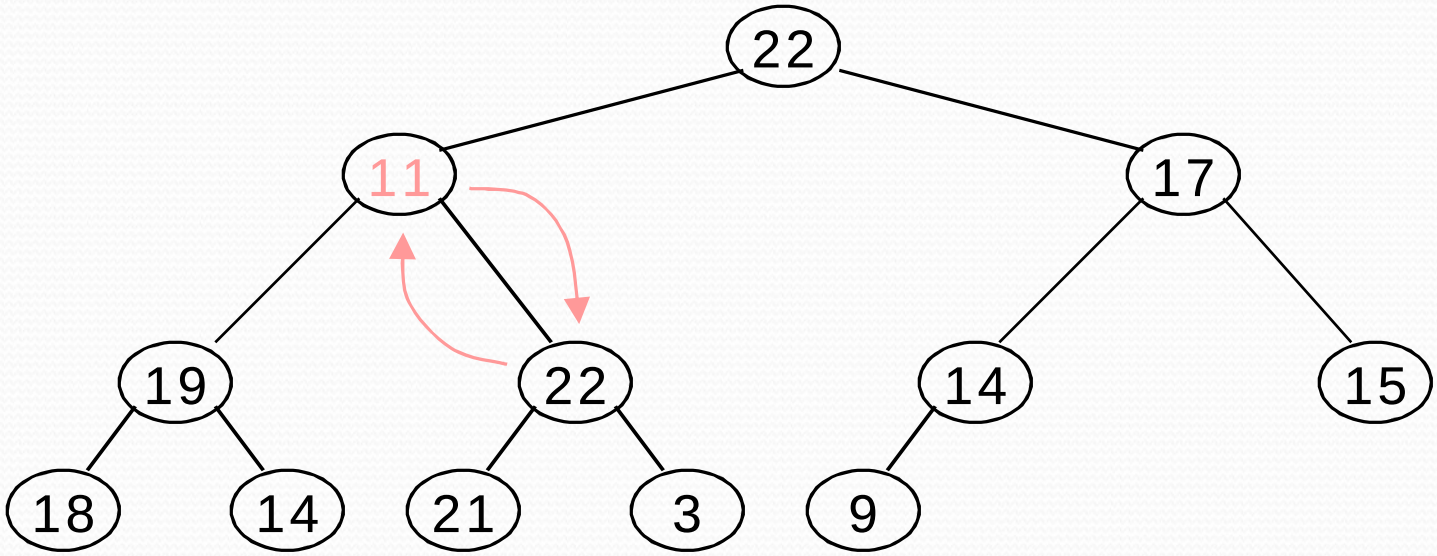
- Our tree is balanced and left-justified, but no longer a heap
- However, *only the root* lacks the heap property



- We can siftUp() the root
- After doing this, one and only one of its children may have lost the heap property

The reHeap method II

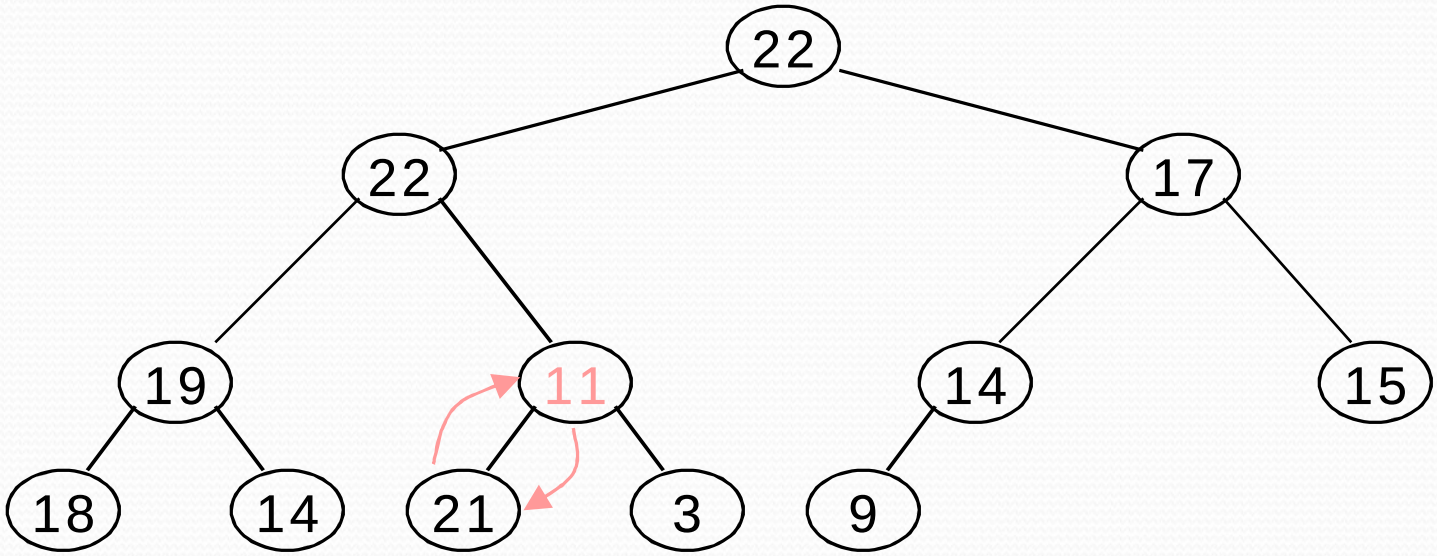
- Now the left child of the root (still the number 11) lacks the heap property



- We can siftUp() this node
- After doing this, one and only one of its children may have lost the heap property

The reHeap method III

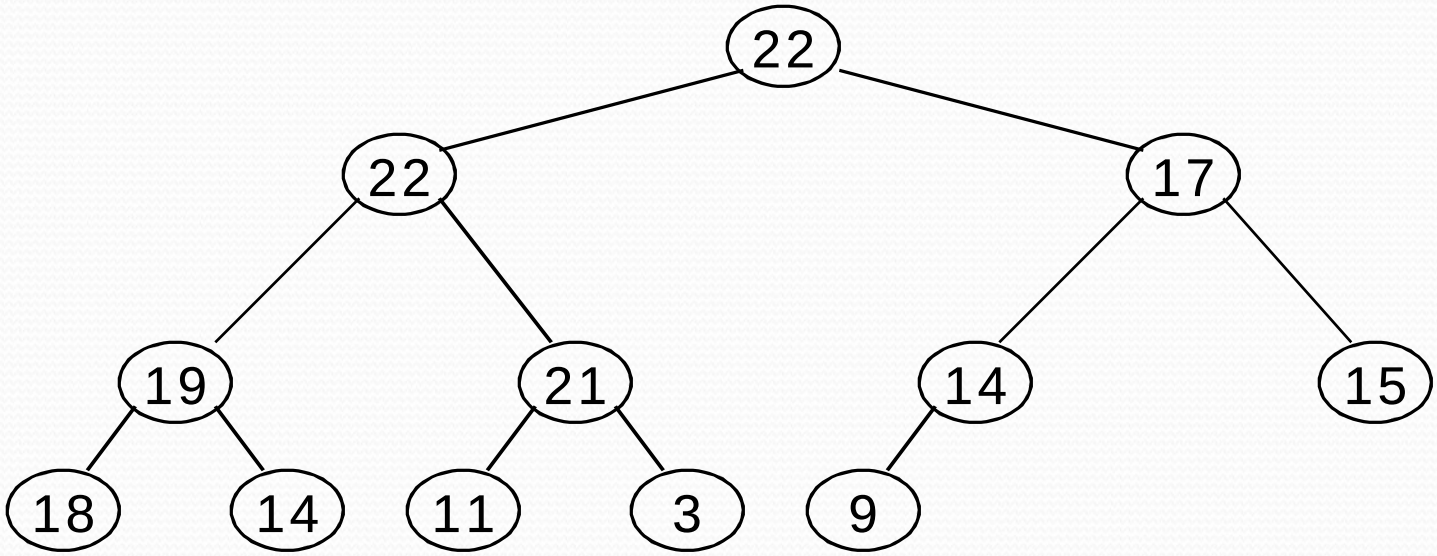
- Now the right child of the left child of the root (still the number 11) lacks the heap property:



- We can siftUp() this node
- After doing this, one and only one of its children may have lost the heap property —but it doesn't, because it's a leaf

The reHeap method IV

- Our tree is once again a heap, because every node in it has the heap property



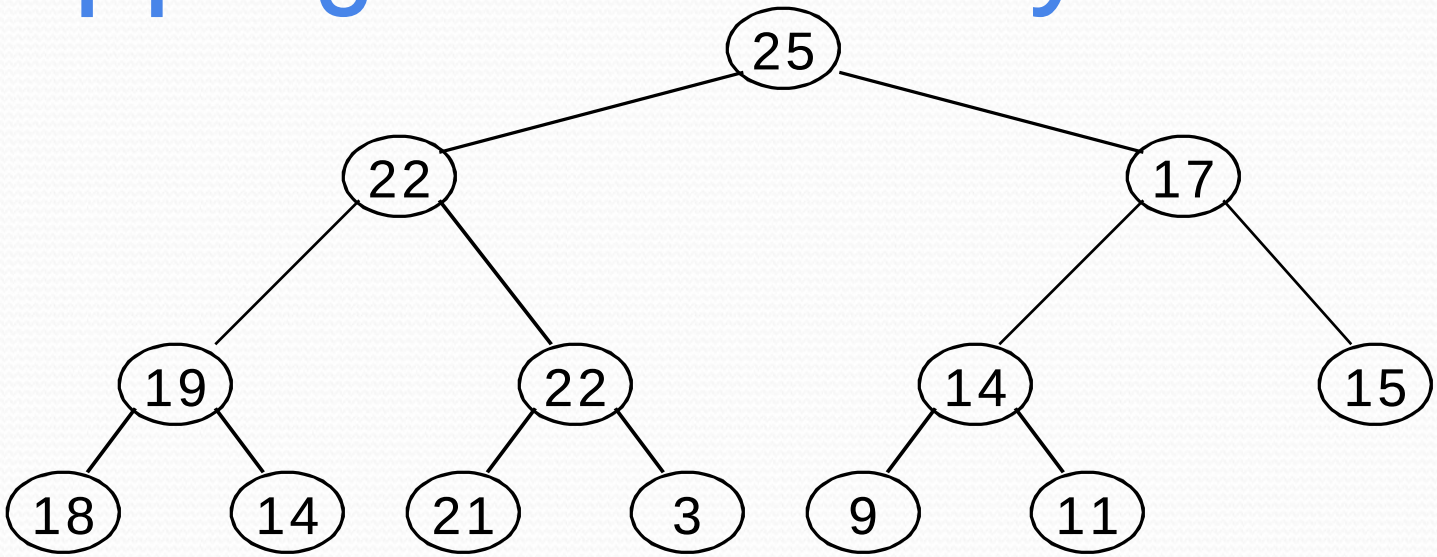
- Once again, the largest (or *a* largest) value is in the root
- We can repeat this process until the tree becomes empty
- This produces a sequence of values in order largest to smallest

Sorting

- What do heaps have to do with sorting an array?
- Here's the neat part:
  - Because the binary tree is *balanced* and *left justified,* it can be represented as an array
  - All our operations on binary trees can be represented as operations on *arrays*
  - To sort:

```
heapify the array;
while the array isn't empty {
    remove and replace the root;
    reheap the new root node;
}
```

Mapping into an array

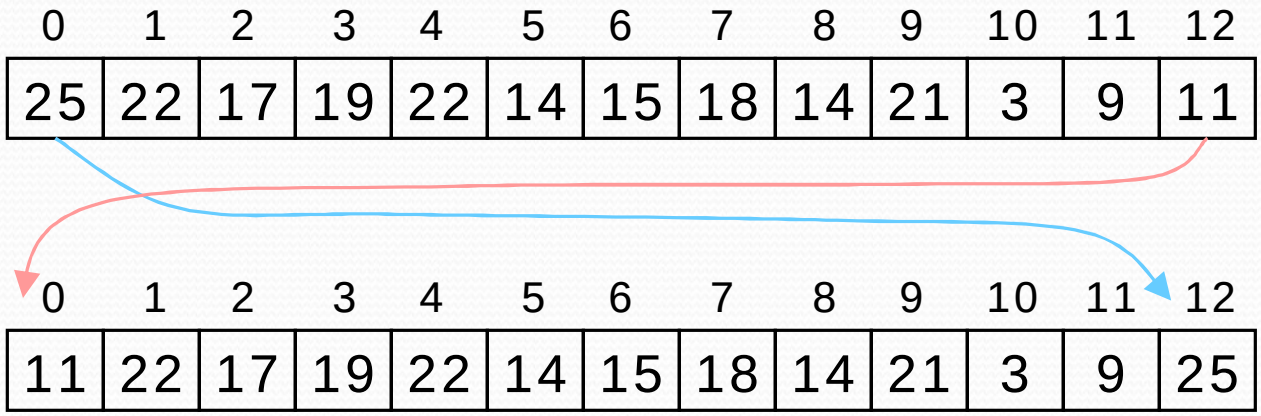


0	1	2	3	4	5	6	7	8	9	10	11	12
25	22	17	19	22	14	15	18	14	21	3	9	11

- Notice:
  - The left child of index i is at index 2*i+1
  - The right child of index i is at index 2*i+2
  - Example: the children of node 3 (19) are 7 (18) and 8 (14)

Removing and replacing the root

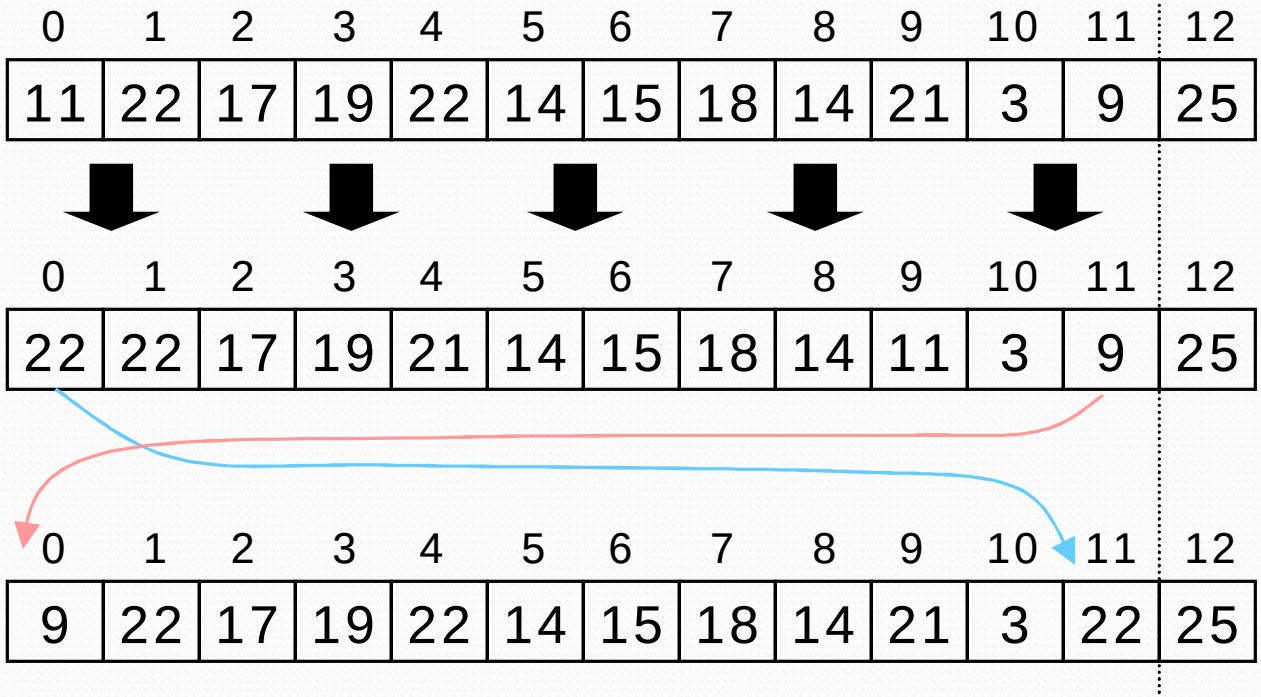
- The “root” is the first element in the array
- The “rightmost node at the deepest level” is the last element
- Swap them...



- ...And pretend that the last element in the array no longer exists—that is, the “last index” is 11 (9)

Reheap and repeat

- Reheap the root node (index 0, containing 11)...



- ...And again, remove and replace the root node
- Remember, though, that the “last” array index is changed
- Repeat until the last becomes first, and the array is sorted!

Analysis I

- Here's how the algorithm starts:

  heapify the array;

- Heapifying the array: we add each of n nodes
  - Each node has to be sifted up, possibly as far as the root
    - Since the binary tree is perfectly balanced, sifting up a single node takes $O(\log n)$ time
  - Since we do this n times, heapifying takes $n*O(\log n)$ time, that is, $O(n \log n)$ time

Analysis II

- Here's the rest of the algorithm:

```
while the array isn't empty {
    remove and replace the root;
    reheap the new root node;
}
```

- We do the while loop n times (actually, $n-1$ times), because we remove one of the n nodes each time
- Removing and replacing the root takes $O(1)$ time
- Therefore, the total time is n times however long it takes the reheap method

Analysis III

- To reheap the root node, we have to follow *one path* from the root to a leaf node (and we might stop before we reach a leaf)
- The binary tree is perfectly balanced
- Therefore, this path is O(log n) long
  - And we only do O(1) operations at each node
  - Therefore, reheaping takes O(log n) times
- Since we reheap inside a while loop that we do n times, the total time for the while loop is n*O(log n), or O(n log n)

Analysis IV

- Here's the algorithm again:

```
heapify the array;
while the array isn't empty {
    remove and replace the root;
    reheap the new root node;
}
```

- We have seen that heapifying takes $O(n \log n)$ time
- The while loop takes $O(n \log n)$ time
- The total time is therefore $O(n \log n) + O(n \log n)$
- This is the same as $O(n \log n)$ time

The End

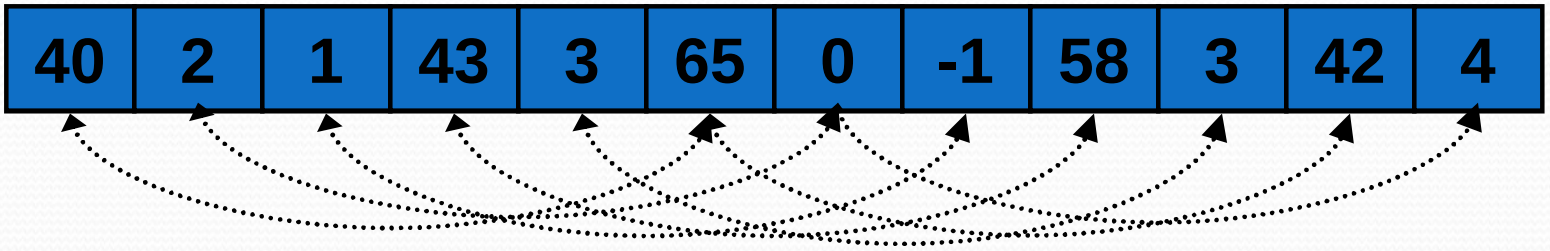
Shell Sort: Idea

Donald Shell (1959): Exchange items that are far apart!

Original:

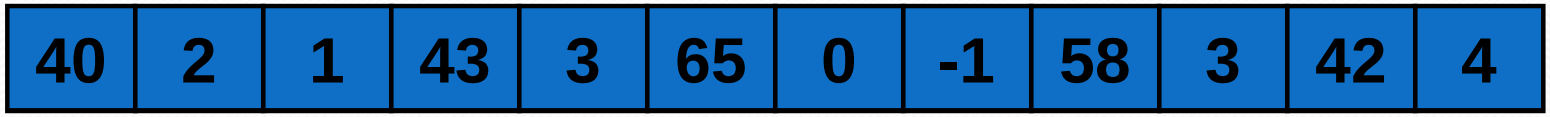
40	2	1	43	3	65	0	-1	58	3	42	4

5-sort: Sort items with distance 5 element:

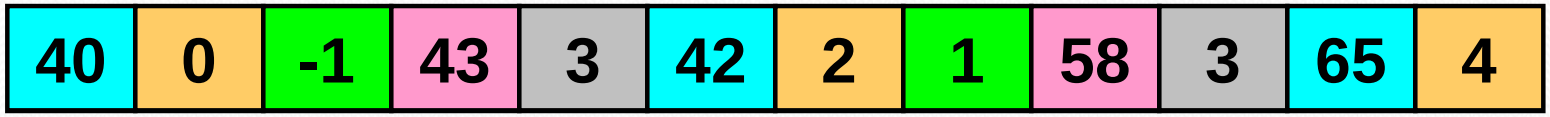


Shell Sort: Example

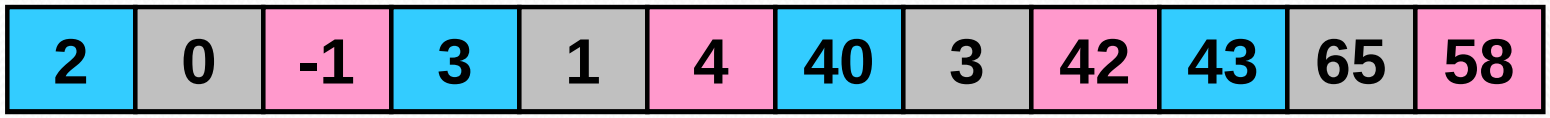
Original:



After 5-sort:



After 3-sort:



After 1-sort:

Shell Sort: Gap Values

- Gap: the distance between items being sorted.
- As we progress, the gap decreases. Shell Sort is also called Diminishing Gap Sort.
- Shell proposed starting gap of N/2, halving at each step.
- There are many ways of choosing the next gap.

Shell Sort: Analysis

N	Insertion Sort	Shellsort		
		Shell's	Odd Gaps Only	Dividing by 2.2
1000	122	11	11	9
2000	483	26	21	23
4000	1936	61	59	54
8000	7950	153	141	114
16000	32560	358	322	269
32000	131911	869	752	575
64000	520000	2091	1705	1249
		$O(N^{3/2})$?	$O(N^{5/4})$?	$O(N^{7/6})$?

So we have 3 nested loops, but Shell Sort is still better than Insertion Sort! Why?

Generic Sort

- So far we have methods to sort integers. What about Strings? Employees? Cookies?
- A new method for each class? No!
- In order to be sorted, objects should be comparable (less than, equal, greater than).
- Solution:
  - use an ***interface*** that has a method to compare two objects.
- Remember: A class that implements an interface inherits the interface (method definitions) = interface inheritance, not implementation inheritance.

Other kinds of sort

- Heap sort. We will discuss this after tree.
- Postman sort / Radix Sort.
- etc.